**2.2 Notes: Calculating Velocity**

* **Velocity** is the change in the position of an object over time. Velocity is a vector, meaning it has a magnitude and direction.
  + The **magnitude** of velocity is its speed.
  + The **direction** of velocity is whichever direction the object is traveling, and can be described using different coordinate systems.

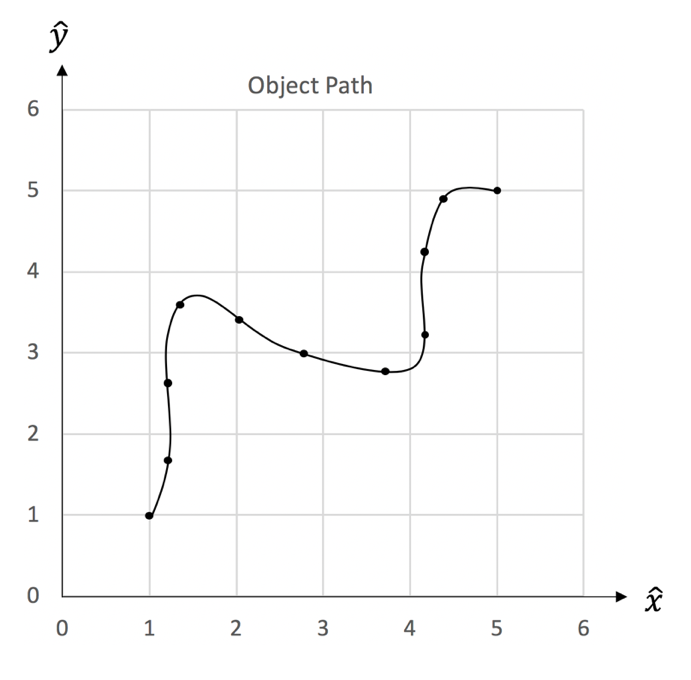
Imagine a car driving in one direction along a street. The car is driving at 2 meters per second in the x-direction. The *speed* of the car is , and we can call the *direction* that the car is moving “”. The *velocity* of the car must be written as a vector, containing both speed and direction:

speed

direction

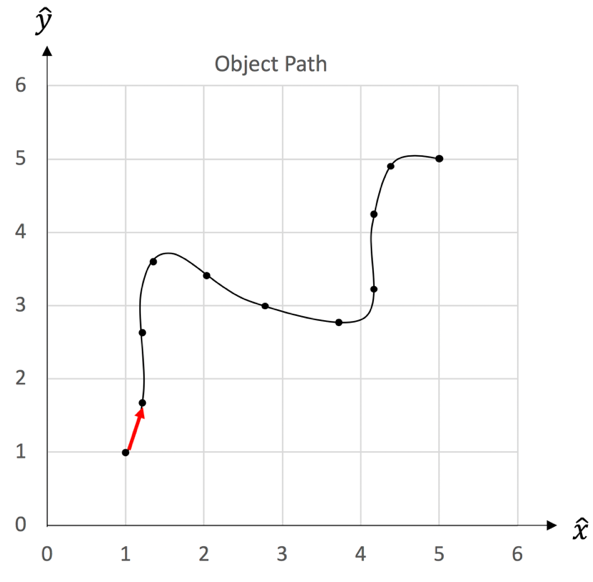
velocity

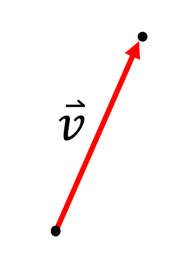
Now, we can imagine an object traveling along a more complicated path, as shown below. The object starts at (x, y) = (1, 1) and ends up at (5, 5) after traveling for ten seconds.

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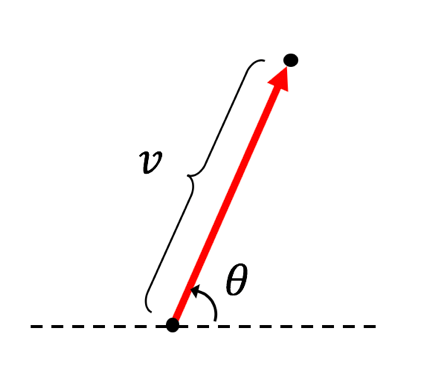
Let’s calculate the object’s velocity over the first second of its motion, shown as the first two black dots on the plot. We can describe the velocity of the object as its change in the position over time.

If we were to draw an arrow between the starting and ending position of the object over a given period of time, that would be the **velocity vector** of the object.

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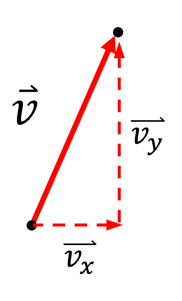
A velocity vector has both a magnitude (size) and direction.

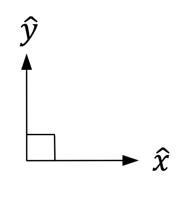
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The **magnitude** of velocity is its speed, , and that is shown as the length of the vector. The speed of the object is the distance it travels over a given period of time.

The **direction** of velocity is whichever direction the object is traveling. This object is traveling at an angle from the horizontal.

An easy way to describe the direction the object is traveling is to break the object’s velocity vector up into components.

We can write the velocity vector as the sum of vectors that point horizontally and vertically:



The horizontal and vertical velocity vectors have their own directions and magnitudes (speeds) as well:

direction

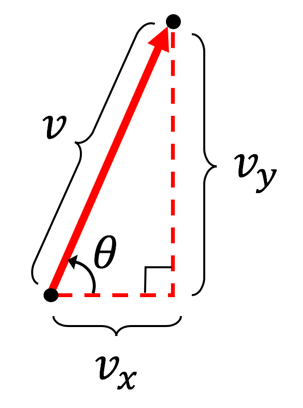
speed

velocity

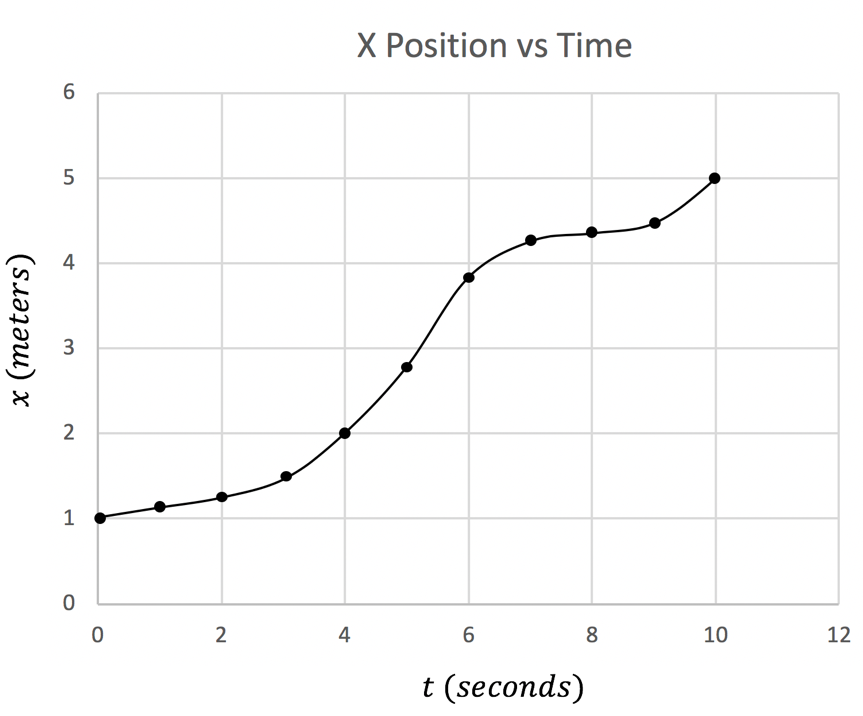
We could write the velocity vector in terms of components as:

This says that the object is traveling at speed in the horizontal direction and speed in the vertical direction.

Because the horizontal and vertical directions are perpendicular to one another, the velocity components form a right triangle. This allows us to solve for certain components of the velocity in terms of other components:



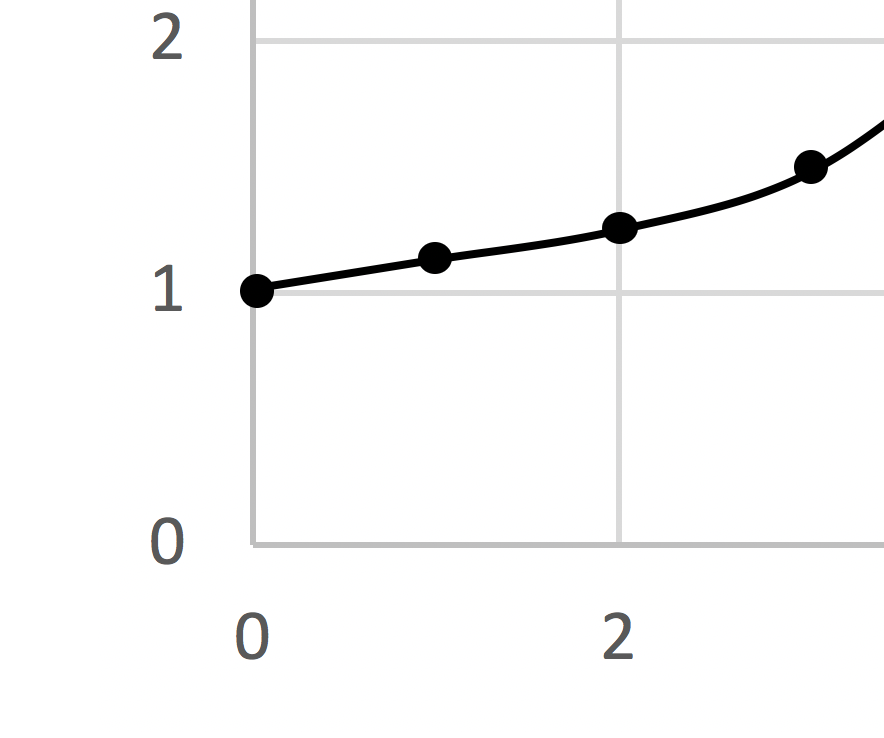
In order to calculate the velocity and speed of the object, we’ll want to calculate the components of the object’s velocity separately. The speed of an object is its change in position over time. Thus, the horizontal and vertical components of the object’s speed over time are given by:

****Let’s begin by calculating It’s helpful to plot the x position of the object over time to make this calculation.

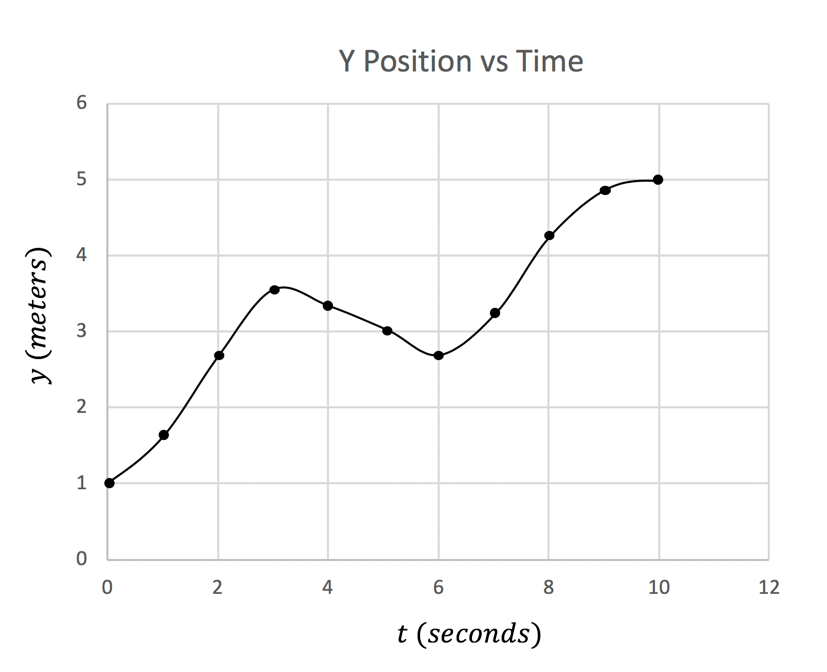
(1, 1.2)

(0, 1)

To calculate during the first second of the object’s motion, we need only use the x positions

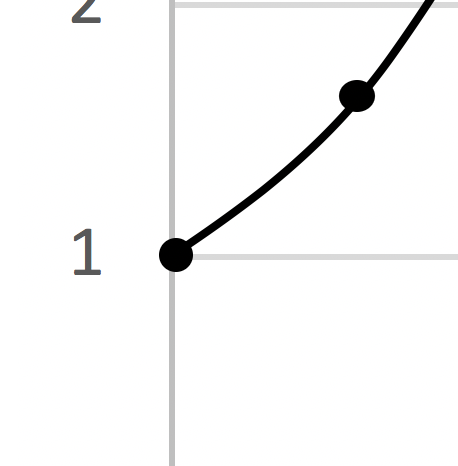
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Thus, during the first second of the object’s motion, we see that the object is moving at 0.2 m/s in the x-direction. Let’s do the same for the y-direction of the motion.



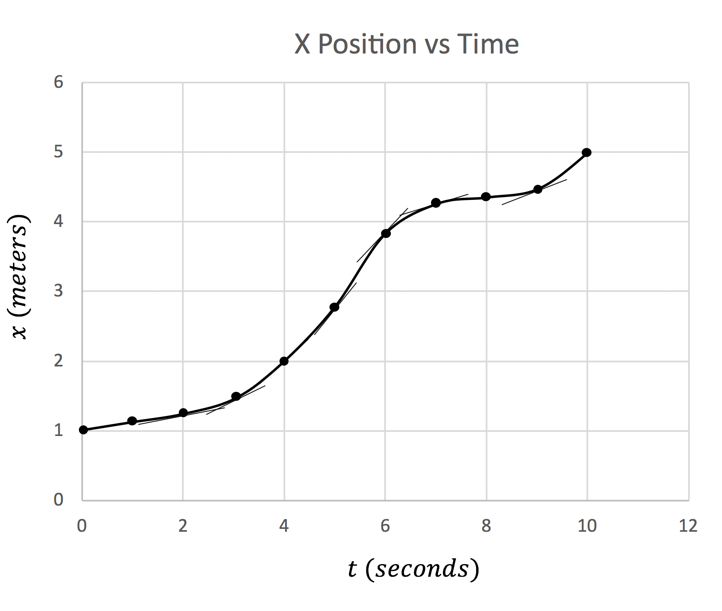
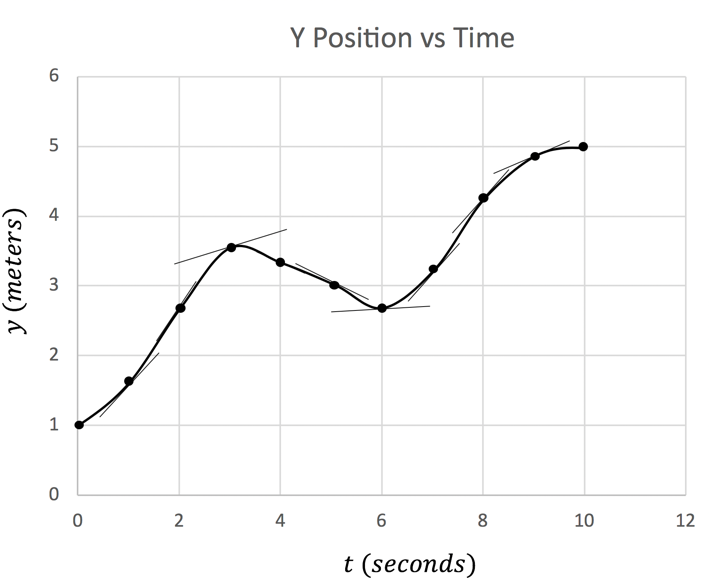
(1, 1.6)

(0, 1)



We can then write the object’s velocity during the first second of its motion as:

We can also notice from the equations for speed that the speed of the object is just the slope (the rise over the run) of the position vs. time plot. Thus, to more accurately estimate the velocity at a given point, one should estimate the slope using two points exactly beside the point in question to increase accuracy. Slop estimates at given points are indicated below.



By determining the slopes of the position vs time plots above at each instant in time, the velocity of the object in the x and y directions can be plotted over time.

